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Quarks, Squarks and Textures.

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Abstract

By studying symmetric mass textures for the up and down quark sectors, and expanding in a small parameter $\lambda \sim sin\theta_C$, bounds are set on entries commonly assumed to vanish. Consequences of a 2 + 1 family structure which can result from horizontal symmetry are examined. Generalizing to squarks, we study suppression of Flavor Changing Neutral Currents by mass degeneracy and/or small mixing angles.

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I. INTRODUCTION

In the Standard Model, the Yukawa sector contains the majority of the free parameters. Though we have now a reasonably good knowledge of the experimental values of these parameters, the flavor theory that may explain the family structure of the fermions and values of the parameters is still a major puzzle. In particular, the strong hierarchy among quark masses and the very nontrivial structure of the CKM matrix remains a conundrum. At a pure phenomenological level, various mass matrix *Ansätze* have been proposed. The most popular Fritzsch *Ansatz* [1] had to be abandoned as we realized that the top-quark mass is above $90GeV$ [2]. However, a modified version [3,4] still holds promise. The Fritzch *Ansatz* uses a mass matrix of three symmetric texture zeros for both the up- and down-mass matrices, while the modified-Fritzch *Ansatz* uses one with two. In the latter case, the zeros are at the $11-$ and $13-(31-)$entries. Having small entries at the locations has been shown to yield favorable relations among the mass and mixing parameters [5]. Recently, the authors of Ref . [6] analyzed all possible symmetric quark mass matrices with the maximal number of texture zeros. They started by assigning to the up (or down) quark mass matrix any of the 6 possible forms of symmetric matrix with an hierarchy of three non-zero eigenvalues and three texture zeros; they then examined admissible solutions with maximal number of texture zeros in the corresponding down (or up) quark mass matrix by fitting the experimental quarks masses and CKM mixing parameters RG-evolved to the GUT scale. Five solutions are listed (we will refer to them as RRR-textures), with five texture zeros and a hierarchical form with matrix entries expressed as leading-order powers of $\lambda = sin\theta_C \sim 0.22$ [7].

On the other hand, interest in the use of horizontal symmetry to derive a phenomenologically viable quark mass texture has been resurrected recently [8–16], partly motivated by the possibility of obtaining simultaneously appropiately constrained squark (soft) mass(-squared) matrices, in the place of an assumed universality condition, for satisfying the relevant FCNC constraints [17,18]. A $SU(2)$ (or $U(2)$) horizontal symmetry with the lighter two

families forming a doublet has then been advocated by some authors [11,13,15–17,19–21]. This $2+1$ family structure has the favorable feature that squark degeneracy among the two families is guaranteed before the breaking of the horizontal symetry. In addition, if this symmetry is gauged, as is desirable, but its breaking goes through a discrete subgroup, possibly dangerous D-term contributions which may lift the squark degeneracy can be avoided and it is possible that a model can be built with all the relevant FCNC constraints satisfied.

Motivated by our SUSY-GUT compatible horizontal symmetry model building [15,16], we will address here some features of the quark and squark mass matrices. In the first part, we derive some more general quark mass textures, using a simple algebraic analysis. New texture patterns obtained contain less texture zeros and are therefore less predictive. This is necessary, however, because the RRR-textures are in general incompatible with vertical unification with a $2+1$ *ansatz*. In the second part, we look at the squark mass matrices and the FCNC contraints from neutral meson mixing, under the perspective of a $2+1$ family structure. Our analysis here is not constrained to GUT-scale. RG-runnings of quark mass ratios are in general small and not important in texture pattern analysis. For the squark masses analysis, we actually consider low energy FCNC constraints. Some discussions and references on the issue is given in our horizontal symmetry model presentation in Ref. [16].

In the quark mass matrix texture analysis below, our objective is to use a simple algebraic analysis to illustrate how the textures are constrained by the mass and CKM parameters, and to derive some more general texture patterns. The analysis allows us to arrive at a wide variety of acceptable texture solutions which are phenomenologically viable. Our solutions are compatible and include as special cases all the five-zero RRR-texture solutions and serve as natural generalizations of them. This leads us to believe that we do not miss any interesting texture pattern. However, no attempt has been made to prove that the textures derived here are the only possible ones under the assumptions, nor do we make such a claim.

II. QUARK MASS MATRIX TEXTURES

We consider a symmetric hierarchical mass matrix given as

$$M = \begin{pmatrix} 0 & x & y \\ x & a & c \\ y & c & 1 \end{pmatrix} \tag{1}$$

where a, c, x and $y \leq \lambda$ (of order $\lambda^n, n \geq 1$). We have thus assumed only one zero in each mass matrix. The entry M_{11} is the one that is most commonly believed to be small. Assuming it is zero here simpifies our analysis. We will later show limits on the entry that can be admitted without upsetting our texture pattern solutions. Starting with the only order one entry M_{33} , we put in the small entries and obtain the three eigenvalues together with the diagonalizing matrix V through a perturbational approach. For all the numbers, *we are interested only in their approximate values as represented by orders in* λ, hence we keep only potentially leading-order terms.

We put in a and x first, and then c and y, the latter as a perturbation to the diagonalized matrix from the former. We have then

$$VMV^\dagger = V_2 V_1 \begin{pmatrix} 0 & x & y \\ x & a & c \\ y & c & 1 \end{pmatrix} V_1^\dagger V_2^\dagger \sim V_2 \begin{pmatrix} -x^2/a & 0 & y - cx/a \\ 0 & a + x^2/a & c \\ y - cx/a & c & 1 \end{pmatrix} V_2^\dagger \sim M(diag) \ , \tag{2}$$

$$V_1^\dagger \sim \begin{pmatrix} 1 & x/a & 0 \\ -x/a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{3}$$

and

$$V_2^\dagger \sim \begin{pmatrix} 1 & 0 & y - cx/a \\ 0 & 1 & c/(1-a) \\ -(y - cx/a) & -c/(1-a) & 1 \end{pmatrix} . \tag{4}$$

Here we note that in the first step, where x is treated as a perturbation, x must be taken smaller than a (*i. e.* x is at least one order higher in λ). We take this assumption here and leave the alternative situation to be handled later. Specifically, we assume $x \leq \lambda^2$ and $x/a \leq \lambda$. The final result is as follows:

$$V^\dagger = V_1^\dagger V_2^\dagger \sim \begin{pmatrix} 1 & x/a & (y - cx/a) \\ -x/a & 1 & c - yx/a \\ -(y - cx/a) & -(c - yx/a) & 1 \end{pmatrix} , \tag{5}$$

$$M(diag) \sim \{ \ -\frac{x^2}{a} - (y - c\frac{x}{a})^2, \ \ a + \frac{x^2}{a} - \frac{c^2}{1-a}, \ \ 1 + \frac{c^2}{1-a} + (y - c\frac{x}{a})^2 \ \} \ . \tag{6}$$

Note that for the mass eigenvalue expressions, Eq.(6), we have chosen to display the principal terms from each entry to the matrix, not just the possible overall leading order terms. For instance, x^2/a cannot be a overall leading order term for the middle eigenvalue as the same term contributes to the smallest eigenvalue.

The result can then be applied to both the up- and down-quark mass matrices. We take $M^u(diag)$ and $V_u^\dagger$ as given by the above equations and $M^d(diag)$ and $V_d^\dagger$ as given by analog equations with a, x, c and y replaced by a', x', c' and y'; with the masses normalized to $m_t = 1$ and $m_b = 1$ respectively. In addition to the expressions for the mass eigenvalues, we have also the elements of the CKM-matrix ($V_{CKM} = V_u V_d^\dagger$); all these parameters can be derived from experimental measurements and expressed in powers of λ [6,10]. Assuming no delicate cancellation of numbers in the expressions of the mass eigenvalues and mixings, at least one term in each must be of the right order in λ. Hence we arrive at the following list of constraints:

1. $x(x/a), (y - cx/a)^2 \leq \lambda^8$;

2. $a, c^2 \leq \lambda^4$;

3. $x'(x'/a'), (y' - c'x'/a')^2 \leq \lambda^4$;

4. $a', c'^2 \leq \lambda^2$;

5. $x/a, x'/a' \leq \lambda$;

6. $c, c' \leq \lambda^2$;

7. $y', c'x'/a', c'x/a, cx'/a' \leq \lambda^3 (\sim \lambda^4)$.

where at least one term in each case must satisfy the equality, rather than inequality. In the last three constraints, which are from the CKM-mixings, we have left out terms whose magnitudes have an upper bound already more strongly constrained by the mass matrices. Combining the conditions [22] then leads to

$$a' \sim \lambda^2, \qquad x' \sim \lambda^3, \qquad a \sim \lambda^4,$$

with the following solutions:

- Case 1

$$M^u \sim \begin{pmatrix} (\leq \lambda^8) & \lambda^6 & \leq \lambda^5 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \leq \lambda^5 & \lambda^2 & 1 \end{pmatrix} \; OR \; M^u \sim \begin{pmatrix} (\leq \lambda^8) & \leq \lambda^6 & \lambda^4 \\ \leq \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad M^d \sim \begin{pmatrix} (\leq \lambda^4) & \lambda^3 & \leq \lambda^3 \\ \lambda^3 & \lambda^2 & \leq \lambda^2 \\ \leq \lambda^3 & \leq \lambda^2 & 1 \end{pmatrix};$$

- Case 2

$$M^u \sim \begin{pmatrix} (\leq \lambda^8) & \lambda^6 & \leq \lambda^4 \\ \lambda^6 & \lambda^4 & \leq \lambda^2 \\ \leq \lambda^4 & \leq \lambda^2 & 1 \end{pmatrix} \; OR \; M^u \sim \begin{pmatrix} (\leq \lambda^8) & \leq \lambda^6 & \lambda^4 \\ \leq \lambda^6 & \lambda^4 & \leq \lambda^2 \\ \lambda^4 & \leq \lambda^2 & 1 \end{pmatrix}, \quad M^d \sim \begin{pmatrix} (\leq \lambda^4) & \lambda^3 & \leq \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \leq \lambda^3 & \lambda^2 & 1 \end{pmatrix}.$$

Note that the M_{11} entries are limits, put in *a posteriori*, that can be allowed without upsetting the solutions. Allowing any M_{11} entry to take the its maximum value to begin with will however modify the constraints.

Alternatively, one can put in c and a first, and then x and y. We have then

$$VMV^\dagger = V_2 V_1 M V_1^\dagger V_2^\dagger \sim V_2 \begin{pmatrix} 0 & x - cy & xc + y \\ x - cy & -c^2 + a & 0 \\ xc + y & 0 & 1 \end{pmatrix} V_2^\dagger \sim M(diag) \qquad (7)$$

$$V_1^\dagger \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & -c & 1 \end{pmatrix} \tag{8}$$

$$V_2^\dagger \sim \begin{pmatrix} 1 & -(x-cy)/(c^2-a) & (xc+y)/(1+c^2) \\ (x-cy)/(c^2-a) & 1 & 0 \\ -(xc+y)/(1+c^2) & 0 & 1 \end{pmatrix} \tag{9}$$

and

$$M(diag) \sim \{ \ \frac{(x-cy)^2}{(c^2-a)} - \frac{(xc+y)^2}{(1+c^2)}, \ \ a - c^2 - \frac{(x-cy)^2}{(c^2-a)}, \ \ 1 + c^2 + \frac{(xc+y)^2}{(1+c^2)} \ \}. \tag{10}$$

Here we aim at alternative texture patterns that need not satisfy $x/a \leq \lambda$. Hence, we consider only the case with $a << c^2$ (at least one higher order in λ). Note that we need also $x/c^2, y/c \leq \lambda$ for the perturbational approximation to be valid. Taking these expressions for $M^u(diag)$ and $V_u^\dagger$ and the previous result for the down-sector to repeat the analysis, we obtain an alternative texture patterns as

- Case 3

$$M^u \sim \begin{pmatrix} (\leq \lambda^8) & \lambda^6 & \leq \lambda^4 \\ \lambda^6 & \leq \lambda^5 & \lambda^2 \\ \leq \lambda^4 & \lambda^2 & 1 \end{pmatrix} \qquad M^d \sim \begin{pmatrix} (\leq \lambda^4) & \lambda^3 & \leq \lambda^3 \\ \lambda^3 & \lambda^2 & \leq \lambda^2 \\ \leq \lambda^3 & \leq \lambda^2 & 1 \end{pmatrix}.$$

Compared with the previous result, one can see easily that taking this alternative approach to M^d (*i. e.* $a' << c'^2$) does not lead to any consistent solution.

Starting from simple assumptions, therefore, we have succeeded in deriving the above hierarchical mass texture patterns [23].

Here we compare our result with the RRR-textures. Unlikely, the latter, we have not go through a detailed numerical analysis. However, we start with a much more general form for the mass matrices and show that the simple algebraic analysis is powerful enough for us

to obtain the various texture patterns, with undetermined coefficients of order unity. The texture patterns are in a sense generalizations of the RRR-textures. Note that in the latter analysis the zeros in general do not have to be exact. For instance, replacing a zero with an entry higher order in λ than all the other entries in the matrix will not upset the solution. We see that without the prior assumption of the existence of many texture zeros, some of the small entries in the mass matrices can actually be much larger than one would expect them to be, from naively applying the RRR-textures. This would be of interest from the model building perspective.

For a detail comparison, first we note that our result gives a down-quark mass matrix always in the form

$$M^d \sim \begin{pmatrix} * & \lambda^3 & * \\ \lambda^3 & \lambda^2 & * \\ * & * & 1 \end{pmatrix} \quad ; \tag{11}$$

while the common structure for all the RRR-textures has the form

$$M^d \sim \begin{pmatrix} * & \lambda^4 & * \\ \lambda^4 & \lambda^3 & * \\ * & * & 1 \end{pmatrix} \quad . \tag{12}$$

A power of λ analysis of the latter form gives

$$M^d(diag) \sim \{ \ \lambda^5, \quad \lambda^3, \quad 1 \ \} \tag{13}$$

instead of the more popular

$$M^d(diag) \sim \{ \ \lambda^4, \quad \lambda^2, \quad 1 \ \} \tag{14}$$

that we used. It can be checked easily that if we started by putting the former $M^d(diag)$ into constraints 3 and 4 in our list, all of our analysis would go through with the only modification in our solutions given by changing M^d to the form of Eq. (11). Recall that $\lambda \sim 0.22$, and order one coefficients are allowed in all the terms. Large coefficients would

easily change the order of λ result. This kind of ambiguity is unfortunately unavoidable in the type of order in λ analysis. The approach is a useful one for obtaining mass matrix *ansätze* or textures [25] but not exact results. To further our comparison, we will assume this alternative M^d solutions for all three cases. Then, the only apparent conflict of our M^d results with the RRR-textures is that the 23/32-entry in case 2 is fixed at λ^2, while the correspondent entry in RRR-textures, if not zero, is given by λ^3. However, there is a large coefficient of 4 from their numerical analysis. Hence a coefficient a bit smaller than one for our case would reconcile the difference. A conflict appears in the limiting form of case 3, which indicates that a six-zero texture pattern is admitted by the naive algebraic analysis, while six-zero cases are ruled out in the RRR-analysis. This particular six-zero pattern remains actually a very popular candidate [26]. Putting in a λ^3 term for the 23/32-entry of M^d while keeping the other zeros, however, does give one of the RRR-textures. Otherwise, the other RRR-textures all fit in with our patterns. Detailed numerical analysis is of course useful to further establishing the viablity of the texture patterns obtained here. Nevertheless, so far as a texture pattern with less zeros is concerned, the extra coefficients definitely give more flexiblility for fitting the experimental parameters. Hence we do expect these texture patterns to be valid, except possibly the six-zero texture.

III. SQUARK MASS MATRICES AND FCNC CONSTRAINTS

Now we turn to the scalar quark sector and look into how a $2+1$ family structure fits into the squark mediated FCNC contraints from neutral meson mixings. First note that the squark mass matrices $\tilde{M}^{u2}$ and $\tilde{M}^{d2}$ are each divided into four 3×3 sub-matrices as

$$\tilde{M}^{u2} = \begin{pmatrix} \tilde{M}^{u2}_{LL} & \tilde{M}^{u2}_{LR} \\ (\tilde{M}^{u2}_{LR})^\dagger & \tilde{M}^{u2}_{RR} \end{pmatrix}, \qquad \tilde{M}^{d2} = \begin{pmatrix} \tilde{M}^{d2}_{LL} & \tilde{M}^{d2}_{LR} \\ (\tilde{M}^{d2}_{LR})^\dagger & \tilde{M}^{d2}_{RR} \end{pmatrix}. \tag{15}$$

The leading contributions to the off-diagonal blocks arise from the trilinear A-terms, while the leading contributions to the diagonal blocks arise from the soft mass terms. The latter dominate over the former, and can generally lead to unacceptably large FCNC-effect in

neutral meson mixing when universality of soft masses is not imposed. Hence they are our subject of concern here [27]. We start by considering the following (diagonal block) mass matrix for general squarks

$$\tilde{M}^2 = \tilde{m}^2 \begin{pmatrix} \tilde{a} & \tilde{x} & (\tilde{c}+\tilde{y})/\sqrt{2} \\ \tilde{x} & \tilde{a} & (\tilde{c}-\tilde{y})/\sqrt{2} \\ (\tilde{c}+\tilde{y})/\sqrt{2} & (\tilde{c}-\tilde{y})/\sqrt{2} & \tilde{b} \end{pmatrix} \tag{16}$$

where $\tilde{a}, \tilde{b}$ are order 1 and $\tilde{c}, \tilde{x}, \tilde{y} \leq \lambda$. Note that order 1 quantities are expected for the diagonal entries as the correspondent mass terms are naturally invariant under any horizontal symmetry. Equality of the first two diagonal entries is dictated by the $2+1$ structure which we are interested in here. The symmetry structure also suggests that any higher dimensional, horizontal symmetry breaking, mass term naturally gives the same contributions to the two entries, and hence lifting in degeneracy of the two mass eigenvalues can be attributed to only the contributions of the non-diagonal terms [28].

We first take a rotation to diagonalize the upper two by two block and add in the rest by a perturbational analysis:

$$\tilde{V}\tilde{M}^2\tilde{V}^\dagger = \tilde{V}_0 V_0 \tilde{M}^2 V_0^\dagger \tilde{V}_0^\dagger \sim \tilde{V}_0 \begin{pmatrix} \tilde{a}-\tilde{x} & 0 & \tilde{y} \\ 0 & \tilde{a}+\tilde{x} & \tilde{c} \\ \tilde{y} & \tilde{c} & \tilde{b} \end{pmatrix} \tilde{V}_0^\dagger \sim \tilde{M}^2(diag) \tag{17}$$

with

$$V_0^\dagger = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}; \tag{18}$$

then

$$\tilde{V}_0^\dagger \sim \begin{pmatrix} 1 & 0 & \tilde{y}/(\tilde{b}-\tilde{a}+\tilde{x}) \\ 0 & 1 & \tilde{c}/(\tilde{b}-\tilde{a}-\tilde{x}) \\ -\tilde{y}/(\tilde{b}-\tilde{a}+\tilde{x}) & -\tilde{c}/(\tilde{b}-\tilde{a}-\tilde{x}) & 1 \end{pmatrix} \tag{19}$$

and

$$\tilde{M}^2(diag) \sim \tilde{m}^2\{ \quad \tilde{a} - \tilde{x} - \frac{\tilde{y}^2}{\tilde{b} - \tilde{a} + \tilde{x}}, \quad \tilde{a} + \tilde{x} - \frac{\tilde{c}^2}{\tilde{b} - \tilde{a} - \tilde{x}}, \\ \tilde{b} + \frac{\tilde{c}^2}{\tilde{b} - \tilde{a} - \tilde{x}} + \frac{\tilde{y}^2}{\tilde{b} - \tilde{a} + \tilde{x}} \quad \}. \tag{20}$$

Note that the difference ($\sim \tilde{b} - \tilde{a}$) between the third and the first or second eigenvalues, is of order one (in $\tilde{m}^2$), while the degeneracy between the first and second eigenvalues is lifted by

$$\Delta\tilde{m}^2_{12} \sim 2\tilde{x} - \tilde{c}^2 + \tilde{y}^2 \tag{21}$$

The other quantity that affects the FCNC is the squark mass mixing matrix which, in the diagonal quark mass basis, is generally expressed as

$$K = V\tilde{V}^\dagger. \tag{22}$$

To be specific, this includes

$$K^d_L = V^d_L\tilde{V}^{d\dagger}_L, \qquad K^d_R = V^d_R\tilde{V}^{d\dagger}_R, \\ K^u_L = V^u_L\tilde{V}^{u\dagger}_L, \qquad K^u_R = V^u_R\tilde{V}^{u\dagger}_R; \tag{23}$$

and the V's and $\tilde{V}$'s are diagonalizing matices for quarks.

$$V^d_L M^d V^{d\dagger}_R = M^d(diag), \qquad V^u_L M^u V^{u\dagger}_R = M^u(diag), \tag{24}$$

and for squarks,

$$\tilde{V}^d_L \tilde{M}^{d2}_{LL} \tilde{V}^{d\dagger}_L = \tilde{M}^{d2}_{LL}(diag), \tilde{V}^d_R \tilde{M}^{d2}_{RR} \tilde{V}^{d\dagger}_R = \tilde{M}^{d2}_{RR}(diag), \\ \tilde{V}^u_L \tilde{M}^{u2}_{LL} \tilde{V}^{u\dagger}_L = \tilde{M}^{u2}_{LL}(diag), \tilde{V}^u_R \tilde{M}^{u2}_{RR} \tilde{V}^{u\dagger}_R = \tilde{M}^{u2}_{RR}(diag). \tag{25}$$

We will however suppress subscripts and superscripts wherever unambiguious.

Taking the hierarchical form of M, as for example given by one of our texture pattern solutions, we have

$$
K = (V_2 V_1) V_0^\dagger \tilde{V}_0^\dagger \sim \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \tilde{c}/\sqrt{2} - y + cx/a \\ -1/\sqrt{2} & 1/\sqrt{2} & \tilde{c}/\sqrt{2} - c \\ -c/\sqrt{2} - \tilde{y} & c/\sqrt{2} - \tilde{c} & 1 \end{pmatrix} \tag{26}
$$

where we have replaced $\tilde{b} - \tilde{a}$ by 1 and keep only the would be leading order terms. However, if we start with

$$
M = \begin{pmatrix} a/2 + x & a/2 & (c+y)/\sqrt{2} \\ a/2 & a/2 - x & (c-y)/\sqrt{2} \\ (c+y)/\sqrt{2} & (c-y)/\sqrt{2} & 1 \end{pmatrix} = V_0^\dagger \begin{pmatrix} 0 & x & y \\ x & a & c \\ y & c & 1 \end{pmatrix} V_0 \ , \tag{27}
$$

then we have

$$
K = (V_2 V_1) V_0 V_0^\dagger \tilde{V}_0^\dagger \sim \begin{pmatrix} 1 & -x/a + cy + \tilde{c}y & \tilde{y} - y + cx/a - \tilde{c}x/a \\ x/a - cy + c\tilde{y} & 1 & \tilde{c} - c + yx/a + \tilde{y}x/a \\ y - \tilde{y} - cx/a & c - \tilde{c} - yx/a & 1 \end{pmatrix} . \tag{28}
$$

Compare the two expressions, one can see that the latter case gives, in general, smaller mixings.

From the perspective of horizontal symmetry, while a hierarchical quark mass matrix, rank one in first order, can be easily enforced, squark mass matrices $\tilde{M}^2_{LL}$ and $\tilde{M}^2_{RR}$ naturally have all their diagonal entries of order one. Unless they are part of the same multiplet, equality of the diagonal squark mass entries does not come naturally. Our choice of diagonal entries is dictated by $2+1$ family structure. The degeneracy is of course lifted by the off-diagonal entries. The first observation here is that when a degeneracy will be lifted by perturbation, the resultant eigenstates are naturally given by a maximal mixing, and hence by the first 2×2 block in Eqs. (18) and (26).

In the quark-squark alignment(QSA) approach [10,18], one gives up the squark mass degeneracy requirement for FCNC suppression. If the quark and squark mass matrices could be almost diagonalized simultaneously, the mixing matrix K would have small off-diagonal elements and hence give the necessary FCNC suppression. However, there is also *no* easy way to obtain such a result from a horizontal symmetry. As shown above, the

degeneracy approach goes to the other extreme, in favor of maximum mixing, for the lighter two generations in our case. After all, in first order form the quark and squark mass matrices are expected to be very different. Is there a way to reconcile this with the QSA? Within the $2+1$ family structure, quark mass matrices of the form given by Eq. (27) seem to give the answer. The first order mixing in K_{12} is removed, as shown in Eq. (28).

The form of the quark mass matrix can be described as democratic in the first two families and hierarchical between them and the third. It takes only a simultaneous rotation of M^u and M^d given by any of the above (phenomenologically viable) hierarchical texture patterns by V_0 to give a pair of matrices in this desired form. We consider it an interesting alternative of *partial* quark-squark alignment. In particular, in the $SU(5)$ unification or a $2+1$ family structure framework, the symmetric nature of M^u and the need to have a relatively large V_{us} make our partial alignment appear as the best option for suppression of K_{12}. The necessary suppression of K_{23} or K_{31} can be easily obtained even without alignment. Quark mass matrices in this form together with the required squark mass matrices can be derived naturally from a Q_{2N} symmetry [16].

Let us complete the analysis by taking a look at the FCNC constraints from the neutral meson mixing [29] and how they can be satisfied within our framework. For instance, constraints from $K-\bar{K}$ and $B-\bar{B}$ mixing on $\tilde{M}^{d2}_{LL}$ can be expressed by an upper bound on

$$(\delta^d_{LL})_{12} = \frac{1}{\tilde{m}^2}(\tilde{m}^2_1 K_{11}K^\dagger_{12} + \tilde{m}^2_2 K_{12}K^\dagger_{22} + \tilde{m}^2_3 K_{13}K^\dagger_{32}) \tag{29}$$

and

$$(\delta^d_{LL})_{13} = \frac{1}{\tilde{m}^2}(\tilde{m}^2_1 K_{11}K^\dagger_{13} + \tilde{m}^2_2 K_{12}K^\dagger_{23} + \tilde{m}^2_3 K_{13}K^\dagger_{33}) \tag{30}$$

respectively, where $\tilde{m}^2_i$ are the three eigenvalues and K is actually $K^d_L = V^d_L \tilde{V}^{d\dagger}_L$ with $\tilde{V}^d_L$ being the unitary matrix that diagonalize $\tilde{M}^{d2}_{LL}$. All the numerical bounds of the type are shown in Table 1. With $K^d_L = V^d_L \tilde{V}^{d\dagger}_L$ given by the form in Eq. (28), the above analysis $(x'/a' \sim \lambda)$ leads to

$$(\delta^d_{LL})_{12} \quad \sim \Delta\tilde{m}^2_{12}(K^d_L)_{12}$$

$$\sim \lambda(2\tilde{x} - \tilde{c}^2 + \tilde{y}^2) \tag{31}$$

which gives the bound $\tilde{x}, \tilde{c}^2, \tilde{y}^2 \leq \lambda^3$. The case for the $(\delta^d_{LL})_{13}$ constraint looks more complicated. However, if we take $\tilde{c} \leq \lambda^2$ and $\tilde{y} \leq \lambda^3$, we would have

$$K^d_L \sim \begin{pmatrix} 1 & \lambda & \leq \lambda^3 \\ \lambda & 1 & \leq \lambda^2 \\ \leq \lambda^3 & \leq \lambda^2 & 1 \end{pmatrix} \tag{32}$$

giving easily $(\delta^d_{LL})_{13} \leq \lambda^3$, for example.

This illustrates how sufficient FCNC suppression can be obtained within this scheme. Details of the various constraints on the parameters in the squark mass matrix of the form given by Eq. (16) is listed in Table 2.

For an explicit application of the kind of algebraic *ansatz* presented here, readers are referred to our $Q_{12} \otimes U(1)$ horizontal symmetry model built along the pattern [16].

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Table Caption.

Table 1: FCNC constraints from neutral meson mixings. The numerical bounds are given as an illustrative set of values (from Ref. [18]), details of which depend on gaugino and squark masses. Necessary suppressions in powers of λ are also given.

Table 2: Details of the constraints on our squark mass matrix parameters. A special point to note is that we impose only the (δ_{LL}) and (δ_{RR}) constraints from Table 1, but not the $\langle\delta\rangle$ ones. For the case of $\left\langle \delta^d_{12} \right\rangle$, it actually leads to a stronger constraint. QSA stands

for quark-squark alignment; partial QSA as described in the text has quark mass matrices of the form given by Eq. (27).

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[22] Notice that the first two constraints, can lead to an apparent solution which has both the two lower eigenvalues (m_u and m_c) generated just from the mixing with the third family. This actually gives a vanishing determinant for the mass matrix. For instance in Eq. (6, the case with y^2 and c^2 as the leading contributions to m_u and m_c respectively. This admits a solution of the form

$$M \sim \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & c \\ y & c & 1 \end{pmatrix}$$

which has zero determinant! This "solution" of course has to be discarded.

[23] During the preparation of the manuscript, the authors came across Ref. [24] which analyzes general texture patterns at low-energy, and their RG-evolution to GUT scale. Their "natural mass matrix" philosophy is similar to that of our approach. The actual analysis and solutions are not the same. Among other things, our texture patterns, with some entries given as inequalities, give more general "natural" patterns.

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[27] We neglect in our analysis FCNC constraints from $\tilde{M}^2_{LR}$; assuming proportionality to the Yukawa couplings, such contraints can be easily satisfied.

[28] Assuming a $2+1$ structure, the squarks of the first two families can be represented by a horizontal doublet Φ. A $\Phi^\dagger\Phi$ term is invariant, allowing the leading order a terms. Further contributions to the terms can come from operators of the form

$$\Phi^\dagger\Phi \left\langle S_1\right\rangle \left\langle S_2\right\rangle .....$$

suppressed by powers of some mass scale. The product of the S_i scalars must be an invariant. In order to contribute to the diagonal terms in the mass matrix, the non-zero horizontal symmetry breaking VEVs therefore must come from states that also form an invariant product. This *naturally* gives identical contributions to $\tilde{M}_{11}$ and $\tilde{M}_{22}$; other possibilities have to base on a more contrived mechanism. We hence stick to the case in our analysis.

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$K-\bar{K}$ mixing	$(\delta^d_{LL})_{12}$	$(\delta^d_{RR})_{12}$	$\left\langle\delta^d_{12}\right\rangle$
upper bound	0.05	0.05	0.006
	λ^3	λ^3	λ^4
$B-\bar{B}$ mixing	$(\delta^d_{LL})_{13}$	$(\delta^d_{RR})_{13}$	$\left\langle\delta^d_{13}\right\rangle$
upper bound	0.1	0.1	0.04
	λ^2	λ^2	λ^2
$D-\bar{D}$ mixing	$(\delta^u_{LL})_{12}$	$(\delta^u_{RR})_{12}$	$\langle\delta^u_{12}\rangle$
upper bound	0.1	0.1	0.04
	λ^2	λ^2	λ^2

Table 1: FCNC constraints from neutral meson mixings. The numerical bounds are given as an illustrative set of values (from Ref. [18]), details of which depend on gaugino and squark masses. Necessary suppressions in powers of λ are also given.

	with partial QSA	without partial QSA
$\tilde{M}^{d2}_{LL}$, $\tilde{M}^{d2}_{RR}$	$\tilde{x} \leq \lambda^3$ $\tilde{c} \leq \lambda^2$ $\tilde{y} \leq \lambda^2$	$\tilde{x} \leq \lambda^4$ $\tilde{c} \leq \lambda^2$ $\tilde{y} \leq \lambda^2$
$\tilde{M}^{u2}_{LL}$, $\tilde{M}^{u2}_{RR}$	$\tilde{x} \leq \lambda$ $\tilde{c}^2 \leq \lambda$ $\tilde{y}^2 \leq \lambda$	$\tilde{x} \leq \lambda^2$ $\tilde{c} \leq \lambda$ $\tilde{y} \leq \lambda$

Table 2: Details of the constraints on our squark mass matrix parameters. A special point to note is that we impose only the (δ_{LL}) and (δ_{RR}) constraints from Table 1, but not the $\langle\delta\rangle$ ones. For the case of $\left\langle\delta^d_{12}\right\rangle$, it actually leads to a stronger constraint. QSA stands for quark-squark alignment; partial QSA as described in the text has quark mass matrices of the form given by Eq. (27).